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STRENGTH EVALUATION FOR A WELDED JOINT WITH A THIN YIELDING INCLUSION OF SMALL SIZE

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UDC 539.375

1. The strength of a welded joint depends on properties of the fusion zone, which may have the form of a thin layer with reduced strength and deformation properties with production defects, including inclusions (see, e.g., [1, 2]). The object of study in this work is a plane model of a welded joint (Fig. 1) which is two half-planes with elasticity moduli E_+ and E_-, and Poisson's ratio ν_+ and ν_- joined through a thin layer of thickness 2h; the E and ν of the layer material either conform with the corresponding elasticity constants of one of the welded materials, or they are intermediate between them (e.g., average). In a certain area the layer is interrupted by an extraneous, relatively yielding, thin inclusion with elasticity modulus E_0. In the Oxy coordinate system shown in Fig. 1 the inclusion occupies the region $|y| \leq h_0 g(x)$, where a is half the inclusion length, h_0 is half the average inclusion thickness ($h_0 \ll a$), and g(x) is a dimensionless shape function for the inclusion whose average value in the section from −a to +a equals unity, i.e., $[g(x)]_a = 1$.

Loading in the model being considered is accomplished at infinity with stress $\sigma_y^\infty = pf(x)$, where p is average stress in the section from −a to +a of axis x, and f(x) is a function of stress distribution inhomogeneity so that $[(f(x)]_a = 1$.

By a relatively yielding inclusion we understand one which leads to positive stress concentration at its ends in the thin layer. The thin layer simulates the fusion zone with reduced (compared with the materials being welded) mechanical properties. Therefore, sources for the start of failure are hypothetically assumed to be parts of the layer adjacent to the ends of the inclusion where there is an unfavorable combination of a high stress level with a low level of strength and deformation properties of the layer metal.

The aim of this work is determination of the critical value of applied load p for small inclusions which are often encountered in engineering practice, and estimation of their

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 146-153, July-August, 1986. Original article submitted June 4, 1984.

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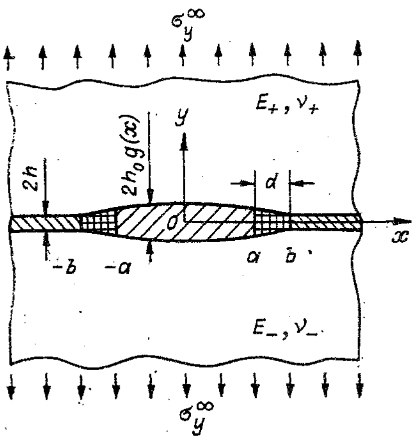

Fig. 1

effect on reducing the strength of the welded joint, which has not been studied very much.

2. Thin yielding inclusions in a homogeneous material have been studied within the framework of the plane problem of linear elasticity theory in a number of works (see, e.g., [3-6]). In these works use has been made of a method involving breaking down boundary conditions for the inclusion boundaries at axis x and reducing the original problem of the inclusion to the corresponding problem of a crack whose opposite sides are loaded by normal stresses $q(x)$, connected with separation (opening) of the crack sides $\delta(x) = v^+(x) - v^-(x)$ by a linear relationship, which for the elastic inclusion in Fig. 1 has the form

$$q(x) = \sigma_0\varphi(x) + E_0\delta(x)/2h_0g(x), \quad |x| < a, \tag{2.1}$$

where $\sigma_0\phi(x)$ are initial stresses in the inclusion presented in the form of their derivative of the average value σ_0 on dimensionless function $\phi(x)$ satisfying the condition $[\phi(x)]_a = 1$ (initial stresses may for example be production stresses which develop as a result of the difference in linear expansion coefficients for the inclusion and the metal matrix).

This method of substituting an inclusion by a crack makes it possible to determine stress intensity factors K_I at the ends of the inclusion and then to use linear fracture mechanics in order to evaluate critical values of applied load p. On the other hand, this approach makes it possible to change over with prescribed radii of curvature for the ends of the inclusion ρ from stress intensity factors to concentration factors $K_t = 1 + 2K_I/p\sqrt{\pi\rho}$ and then to use classical strength theory in order to evaluate critical values of p. However, it is easy to demonstrate that both of these approaches applied to an inclusion with small characteristic dimensions do not give correct results. For example, it follows from [4] for an elliptical inclusion with a shape function $g(x) = (4/\pi)\sqrt{1 - x^2/a^2}$ ($|x| \leq a$) that linear fracture mechanics only operates with $a > K_{Ic}^2/\pi\sigma_t^2\,(1 - k)^2$, where K_{Ic} and σ_t are material fracture toughness and strength at the end of the inclusion, and the effective stiffness of the inclusion $k = (\pi/4)(1 - \nu^2)(a/h_0)(E_0/E)$ should be less than unity, but the concentration factor $K_t = 1 + (\pi/4)(1 - k)(a/h_0)$ calculated from K_I does not depend on inclusion dimensions a and h with a proportional change in them, and it does not make it possible to consider the scale factor.

The difficulties mentioned are similar to those encountered in crack theory, where with the aim of overcoming them the series of approaches suggested is based on considering nonlinear effects in material behavior at the crack ends; in particular, crack models with thin zones of irreversible strains. Instead of an original crack with length 2a, an imaginary crack with increased length 2b is studied in which the resistance of the selected material is replaced by the action of stresses $q(x)$ applied to its opposite sides. In the Leonov–Panasyuk approach [7] stress $q(x)$ is assumed to be constant and is governed by the strength of a thin zone. The local failure criterion in [7] is assumed to be reaching a crack opening at points $x = \pm a$ of a certain critical value δ_c, which is assumed to be a material constant. Unfortunately, this approach may not entirely overcome the limitations on crack length since in order for it to be correct it is necessary that 2a is at least less than δ_c. For application to finer defects it is apparently necessary to give up the assumption that δ_c is a material constant, and to assume that it depends on defect size.

The least complication of the Leonov–Panasyuk approach is the assumption about the presence of a linear relationship between stress q(x) and separation δ(x) of imaginary crack sides in supplementary areas. In the case when it is assumed that q(x) decreases with increasing δ(x); for example, by the equation

$$q(x) = \sigma_{\text{п}} - M\delta(x)/2h, \tag{2.2}$$

where σ_t and M are weakened material strength and modulus in the thin zone, then as was first demonstrated in application to large cracks [8], qualitatively new results are obtained displaying the absence of a necessity for prescribing any failure criterion at the start. Some additional information about this approach is given in [9]. The possibility of using this approach for a small crack has been demonstrated in [10].

In present work relationship (2.2) is used in order to describe the behavior of areas of the fusion zone adjacent to the ends of a thin inclusion, which assumes two things. First, in the vicinity of the inclusion ends there is at first such a density of microdefects that their further development during deformation reduces the strength of the fusion zone to a greater degree than it may be increased as a result of strengthening for continuous (undamaged) material between microdefects. Second, during deformation under special loading conditions, loading of microdefects may proceed stably up to complete material separation along the fusion zone, which corresponds to a smooth reduction in the strength of this material from the original ultimate strength σ_t to a zero value, i.e., its deformation loss of strength. An experimental study of the effect of deformation loss of strength for the fusion zone is possible, but only with very rigid loading conditions and control for displacements δ of its boundaries (see, e.g., [11]). Under actual loading conditions for standard specimens in standard test machines, sections of strength loss in deformation diagrams for the layer, due to excess energy applied to it (compared with that which it may absorb), cannot be entirely stably realized, and it breaks before its strength q(x) decreases to zero. Nonetheless, it is necessary to draw from the overall deformation diagram, as is potentially possible, that there corresponds an increase of δ in (2.2) up to a limiting value $\delta_p = 2h\sigma_t/M$ with which there is a return of stress q to zero. The degree of actual realization of this section of the deformation diagram may be found in the course of solving the problem on the basis of determining conditions for the existence and uniqueness of its solution by the method in [8, 9]. Difficulties of experimental study of the strength loss section and its modulus M may be avoided by means of the well-known approach (see, e.g., [12, 13]) when the area beneath the overall deformation diagram equals energy 2γ expended in forming two new separation surfaces, which in turn is connected with fracture toughness K_{Ic}. For (2.2) this approach gives an estimate

$$h/M = (1 - \nu^2)(K_{Ic}/\sigma_t)^2/E, \tag{2.3}$$

where K_{Ic} is fracture toughness for the fusion zone of a welded joint.

3. In order to solve the problem being considered for an actual inclusion of length 2a together with zones of layer weakening, it is substituted by an imaginary crack of length 2b = 2a + 2d with application to its edges of loads S(x) = pf(x) − q(x). The problem is assumed to be symmetrical relative to the y-axis, which makes it possible to limit consideration to one half-plane $x \geq 0$. Loads q(x) are found in accordance with (2.1) with $0 \leq x \leq a$ and with (2.2) with $a \leq x \leq b$. From solution of the problem for a crack at the interface of two elastic materials it is well known that existence of a difference in elasticity properties for the composite plane leads to a physically incorrect phenomenon of stress and strain oscillation at the crack ends [14, 15]. However, these phenomena are concentrated in a very small area of the crack tips, and in order to obtain a physically correct solution they may be ignored (see, e.g., [12]). In this case the expression for crack side separation relative to dimensionless variable ξ = x/b is obtained from the results in [15] in the form

$$\delta(\xi) = \frac{4m_1 b}{\pi E_1}\int_0^1 S(t)K(\xi, t)\,dt, \quad 0 \leqslant \xi \leqslant 1, \tag{3.1}$$

where $K(\xi, t) = \ln|(\sqrt{1-\xi^2} + \sqrt{1-t^2})/(\sqrt{1-\xi^2} - \sqrt{1-t^2})|$.

$$\frac{2m_1}{E_1} = \frac{m_+(1+\nu_-)}{E_-(1+\nu_+)} + \frac{m_-(1+\nu_+)}{E_+(1+\nu_-)},$$

$m_{\pm} = 1 - \nu_{\pm}^2$ with plane strain and $m_{\pm} = 1$ with a generalized plane stressed state.

Substitution in (3.1) of relationships (2.1) and (2.2), taking account of (2.3), makes it possible to obtain an integral equation relating to dimensionless displacements $w(\xi) = \delta(\xi)(\pi E_1/4m_1\sigma_t a)$:

$$w(\xi) + \frac{b}{a}\left[k_0 \int_0^{a/b} w(t)\frac{K(\xi, t)}{g(t)}\,dt - a_0 \int_{a/b}^{1} w(t)K(\xi, t)\,dt\right] = \tag{3.2}$$
$$= -F_0(\xi) + (p/\sigma_t)F_1(\xi) - (\sigma_0/\sigma_t)F_2(\xi),\ 0 \leqslant \xi \leqslant 1,$$

where

$$F_0(\xi) = \frac{b}{a}\int_{a/b}^{1} K(\xi, t)\,dt;\ F_1(\xi) = \frac{b}{a}\int_0^1 f(t)K(\xi, t)\,dt;$$
$$F_2(\xi) = \frac{b}{a}\int_0^{a/b} \varphi(t)K(\xi, t)\,dt;$$

$k_0 = 2m_1E_0a/\pi E_1h_0$ is inclusion effective stiffness; $a_0 = 2a/\pi I_c$ is inclusion effective size; $I_c = (1 - \nu^2)E_1K_{Ic}^2/m_1E\sigma_t^2$ is relative crack resistance of the layer.

In order to establish the correspondence betwen unknown dimension $d = b - a$ for the weakened zone and load p, it is necessary to add to Eq. (3.2) a condition for the smoothness of imaginary crack side closing, and in fact $d\delta(x)/dx = 0$ with $x = b$, which, taking account of (2.1)-(2.3) and (3.1), relative to $w(\xi)$ is written as

$$k_0\int_0^{a/b}\frac{w(t)\,dt}{g(t)\sqrt{1-t^2}} - a_0\int_{a/b}^{1}\frac{w(t)\,dt}{\sqrt{1-t^2}} = -\int_{a/b}^{1}\frac{dt}{\sqrt{1-t^2}} + \tag{3.3}$$
$$+\frac{p}{\sigma_t}\int_0^1\frac{f(t)\,dt}{\sqrt{1-t^2}} - \frac{\sigma_0}{\sigma_t}\int_0^{a/b}\frac{\varphi(t)\,dt}{\sqrt{1-t^2}}.$$

Stresses $q(\xi)$ in a layer with $x \geq b$ ($\xi \geq 1$) in the arrangement of the problem being considered are expressed by the equation

$$q(\xi)/\sigma_t = \frac{p}{\sigma_t}f(\xi) + \frac{2}{\pi}\xi\sqrt{\xi^2-1}\left\{-\int_{a/b}^{1}\frac{dt}{\sqrt{1-t^2}(\xi^2-t^2)} + \right. \tag{3.4}$$
$$+\frac{p}{\sigma_t}\int_0^1\frac{f(t)\,dt}{\sqrt{1-t^2}(\xi^2-t^2)} - \frac{\sigma_0}{\sigma_t}\int_0^{a/b}\frac{\varphi(t)\,dt}{\sqrt{1-t^2}(\xi^2-t^2)} -$$
$$\left. -k_0\int_0^{a/b}\frac{w(t)\,dt}{g(t)\sqrt{1-t^2}(\xi^2-t^2)} + a_0\int_{a/b}^{1}\frac{w(t)\,dt}{\sqrt{1-t^2}(\xi^2-t^2)}\right\}.$$

A solution of (3.2) with an arbitrary right-hand side does not always exist. Finding conditions for its existence is connected with finding nontrivial solutions for the corresponding homogeneous integral equation. These conditions depend on three dimensionless parameters: k_0, a_0, and b/a. Function $K(\xi, t)$ for variables ξ, t ($0 \leq \xi \leq 1$, $0 \leq t \leq 1$) is determined positively. Therefore, with $b/a = 1$, $k_0 > 0$, $g(t) > 0$ a solution of integral Eq. (3.2) exists and it corresponds to the elastic solution. An increase in the ratio b/a with fixed $k_0 > 0$ and $a_0 > 0$ leads to a reduction in the contribution of the first integral in the left-hand part of (3.2) and to an increase in the contribution of the second integral with a minus sign. Finally, the ratio b/a inevitably reaches a certain value λ_1, which is the least positive characteristic number at which nontrivial solutions develop for the homogeneous equation, and solutions do not exist for the original inhomogeneous integral

Eq. (3.2) with an arbitrary right-hand side of it. Subsequently, achievement of a value of λ_1 for the ratio b/a is treated according to [8, 9] as the onset of a critical condition corresponding to the start of a rapid failure process.

Solution of inhomogeneous Eq. (3.2) with $b/a \neq \lambda_1$ may be accomplished numerically by using a computer. For this purpose it is necessary to approximate all integrals by finite sums by one or another quadratic equation, and to approximate Eq. (3.2) itself by a set of linear algebraic equations. The value of λ_1 achieved by the ratio b/a in this way is fixed by converting the determinant of the approximation system to zero. The following series of calculations is convenient. Values of parameters k_0 and a_0 which are of interest are prescribed, and the ratio b/a is assumed at first to equal unity. Solution of the approximation set of linear algebraic equations (3.2) is accomplished by a Gauss method, which makes it possible to calculate its determinant and simultaneously to obtain three solutions $w_0(\xi)$, $w_1(\xi)$ and $w_2(\xi)$ individually for each term of the right-hand part $F_0(\xi)$, $F_1(\xi)$ and $F_2(\xi)$. From these solutions a linear combination $w(\xi) = w_0(\xi) + (p/\sigma_t)w_1(\xi) + (\sigma_0/\sigma_t)w_2(\xi)$ is formed whose substitution in condition (3.3) makes it possible to obtain an expression for p/σ_t. With given values of k_0, a_0, b/a, and σ_0/σ_t, it is possible to calculate parameter p/σ_t, after which solution $w(\xi)$ becomes completely known. Stresses $p(\xi)$ are calculated from it by Eq. (3.4) with $\xi \geq 1$ and by (2.1) and (2.2) with $0 \leq \xi \leq 1$, which may be rewritten in the form

$$q(\xi)/\sigma_t = (\sigma_0/\sigma_t)\varphi(\xi) + k_0 w(\xi)/g(\xi), \; 0 \leqslant \xi < a/b; \tag{3.5}$$

$$q(\xi)/\sigma_t = 1 - a_0 w(\xi), \; a/b < \xi \leqslant 1. \tag{3.6}$$

At this point the first step of the calculation is complete. Then, step by step, more values of the ratio b/a are prescribed and calculation each time is carried out anew until the determinant of the approxiating system does not change sign. The $q(\xi)/\sigma_t$ and p/σ_t corresponding to this instant are considered to be critical, leading to rapid weld-joint failure.

4. In this work specific calculations were carried out for the case of $f(x) \equiv 1$, $\sigma_0/\sigma_t = 0$, $g(x) \equiv 1$. The dimensionless range of integration from zero to unity was broken down into forty equal sections, in each of which the function $w(\xi)$ sought was considered to be constant, and the integral was taken in closed form. Ratio a/b was reduced with a step of 0.025, which corresponded to an increase in d/a with an increasing step. Two values of effective inclusion size (a_0 = 1 and 2) and five values of effective stiffness coefficient (k_0 = 0, 1, 2, 4, 8) were chosen for the calculations.

Shown in Fig. 2 are curves for the distribution of relative stresses $q(x/a)/\sigma_t$ in the case of $a_0 = 1$ and $k_0 = 2$ (1 relates to $p = 0.48\ \sigma_t$ (with b/a = 1.2), and 2 relates to $p = 0.83\ \sigma_t$ (with b/a = 1.6)]. It can be seen that stresses q are at a maximum and equal to σ_t in the end of the weakened zone at a certain distance d = b − a from the edge of the inclusion. With x > b they decrease rapidly, tending towards the value of applied load p. They also decrease on approach to the edge of the inclusion due to material strain weakening. At the same time, q is at a maximum in the center of the inclusion, where σ_t are not exceeded. On approaching the edge of the inclusion they also decrease. With x = a stresses q undergo a break, and from the direction of the inclusion they are less than from the direction of the layer. The magnitude of the break depends on p, and it decreases with an increase in p. With other values of factors k_0 and a_0 the picture of q distribution remains qualitatively the same as in Fig. 2, only changing quantitatively.

The correlation of p/σ_t with size of the weakened zone d/a = b/a − 1 is illustrated in Fig. 3 with $a_0 = 1$ and k_0 = 8; 4; 2; 0 (lines 1-5). Marked with crosses are final points on the curve corresponding to a return of the determinant of the approximating system to zero, which characterizes the onset of a critical condition. It can be seen that in spite of local weakening of the layer close to the inclusion ends, global curves p–d in Fig. 3, describing behavior of a welded joint with an inclusion as a complete system, have only an ascending branch. This is connected with the fact that in (3.2) parameter p is in the right-hand part and therefore it is assumed to be prescribed; i.e., in point of fact, a condition of ideally soft loading to infinity is assumed. For the condition of ideally rigid loading to infinity, instead of p in the right-hand part of (3.2) it would be necessary to substitute its expression from (3.3) and to transfer the terms obtained relating to $w(\xi)$ to the left-hand part of (3.2), thus excluding the possibility of controlling load. As a result of this, an

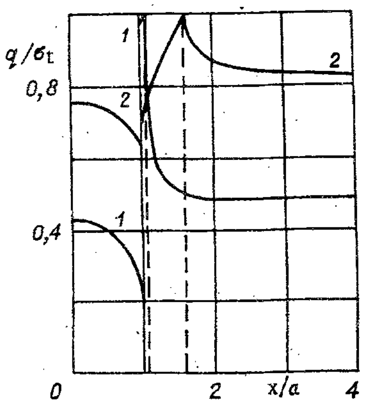

Fig. 2

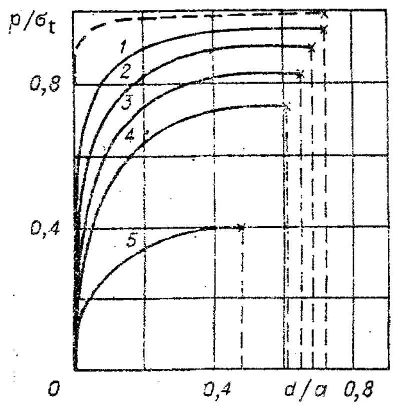

Fig. 3

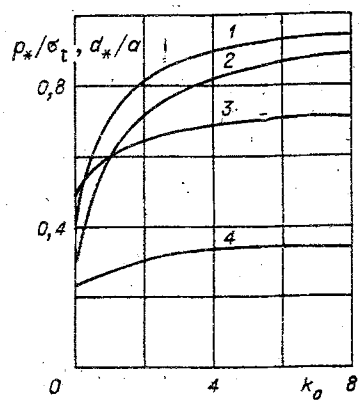

Fig. 4

TABLE 1

k_0	a_0	w_*	Δq_*
0	1,0	0,876	0,124
	2,0	0,498	0,004
1,0	0,1	0,824	0,094
	1,0	0,487	0,026
	2,0	0,333	0,001
8,0	1,0	0,110	0,009
	8,0	0,100	0

integral equation would be obtained with another core and the determinant of the corresponding approximation system would change sign a little later; i.e., curves in Fig. 3 would have a continuation to points marked with crosses. This case of loading is based on facts, although without particular accentuation its application to small cracks was discussed in [10]. From the results of this work it follows that loss of stability for the solution under these loading conditions does not occur, at least while the material in the vicinity of the crack is completely not weakened. In this way, load p after passing a maximum decreases gradually. Since this case of loading is more dangerous, it is not considered here in detail.

The final result of these studies is critical values of p_*/σ_t and relation of them to critical value of d_*/a for the size of the weakened zone whose dependences on k_0 are presented in Fig. 4, where curves 1 and 2 are dependences of d_* on k_0 for $a_0 = 1$ and 2, and 3 and 4 are dependences of d_* on k_0 with the same values of a_0. From the curves provided it can be seen that critical values of p_* and d_* are at a minimum for cracks ($k_0 = 0$), and they gradually increase with an increase in inclusion stiffness. The rate of increase for critical values is at a maximum with small k_0, and then it gradually slows down. Critical loads p_* in the range of values of k_0 being considered nowhere exceed ultimate strength σ_t for the layer in the absence of an inclusion, although they approach it from below with an increase in k_0. It is also noted that k_0 affects p_* to a greater extent than d_*, and conversely a_0 affects d_* to a greater extent than p_*.

5. In this work for calculations the case $q(x) \equiv 1$ was selected corresponding strictly speaking to an inclusion of cosntant thickness. However, as is easily understood, this approach remains qualitatively the same as for inclusions with variable thickness $2h_0g(x)$, where $g(x)$ is a limiting positive function changing weakly along the inclusion length. In addition, the case $g(x) \equiv 1$ may be considered as a first approximation for a thin inclusion of arbitrary shape. In this way (3.2), with any values of k_0 and a_0, always has a completely defined range of possible change in ratios b/a from 1 to λ_1 in which a solution exists and is unique, and the critical load p_* corresponding to λ_1 is less than σ_t and it depends both on stiffness and on inclusion dimensions. All of this points to the advantages of this

approach compared with the well-known methods of elasticity theory and linear fracture mechanics.

Given in Table 1 is dimensionless separation of imaginary crack sides at point $x = a$ in the critical condition which is designated in terms of w_* for certain values of k_0 and a_0. From (3.5) and (3.6) with $w = w_*$ it is possible to calculate breaking stress Δq_* between the layer and the inclusion at point $x = a$. Calculations are also given in Table 1 which indicate that $\Delta q_* > 0$; i.e., stresses from the direction of the layer in the critical condition are greater than from the direction of the inclusion. Since in the inclusion itself stresses are greater than zero, then this means that relationship (2.2) is never realized up to the end (i.e., up to $\delta = \delta_p$, when $q = 0$). The critical condition sets in before the layer material is completely weakened. Inequality $\Delta q_* > 0$ is equivalent to inequality $w_* \le 1/(a_0 + k_0)$. Changing over to dimensional values we obtain $\delta_* \le 2m_1(\sigma_t/E_1) I_c [1 + m_1(E_0/E_1)I_c/h_0]$. Whence it can be seen that with a reduction in inclusion size h_0 critical opening δ_* decreases, tending towards zero. Since Δq_* with decreasing a_0 grows rapidly, then the actual reduction of δ_* proceeds even more rapidly. Thus, the limitation indicated in part 2 on defect size due to δ_c in the given model is overcome as a result of a reduction in δ_*.

The results obtained in this work are extended to the more general case of $0 < \sigma_0 < \sigma_t$, $\phi(t) \equiv 1$, which may be treated as linear strengthening of an inclusion for yield strength σ_0 with strengthening modulus E_0. This in no way affects d_*, since σ_0 is only present in the right-hand part of (3.2). Therefore, curves 3 and 4 in Fig. 4 retain their point with the arbitrary ratio $\sigma_0/\sigma_t < 1$. As far as q and p are concerned, it is then easy to understand from the right-hand part of (3.2) that they depend linearly on σ_0/σ_t:

$$q = \sigma_0 + (1 - \sigma_0/\sigma_\text{п})q_0,\ p = \sigma_0 + (1 - \sigma_0/\sigma_\text{п})p_0,$$

where q_0 and p_0 are values of q and p with $\sigma_0 = 0$. Hence, it follows that with an increase in the ratio σ_0/σ_t there is equalization of forces and stresses normalized for σ_t, which tends everywhere towards a constant value equal to σ_0/σ_t, with σ_0/σ_t tending towards unity. Given in Fig. 3 as an example is the dependence of p on d with $a_0 = 1$, $k_0 = 8$, and $\sigma_0/\sigma_t = 0.8$. It can be seen that the weakened zone develops in the instant of reaching with force p a value σ_0, and when σ_0 is close to σ_t the weakened zone does not develop to a critical state ($p_* \approx \sigma_t$). The case of $\sigma_0 > \sigma_t$ contradicts the statement of the problem considered here. Thus, consideration of $\sigma_0 \ne 0$ leads to an increase in critical loads p_* with retention of d_*, and with a conservative approach it is possible to assume that $\sigma_0 = 0$.

For convenience the results obtained given in this work for critical load may be described with an error not more than 1% by the relationship

$$p_*/\sigma_t = \sigma_0/\sigma_t + (1 - \sigma_0/\sigma_t)/\sqrt{1 + (\pi^2/2)\,a_0/[1 + (\pi^2/2)\,k_0]},$$

in a particular case ($k_0 = 0$, $\sigma_0/\sigma_t = 0$) coinciding the well-known relationship for crack theory (e.g., see [16]) obtained from the energy variation principle assuming proportionality for crack opening and applied load. With $a_0 \ll 1$, from the relationship suggested it follows that $p_* \approx \sigma_t$. In the case of infinitely thin inclusions $a/h_0 \gg 1$ and $k_0 \gg 1$ it is found that the critical load ceases to depend on inclusion length and it is governed b geometry only for its thickness.

In conclusion it is noted that in a real deformation curve flow and strengthening sections preceding the weakening section may be of considerable importance. However, it is evident that with the same area beneath the deformation curve an increase in it of the proportion of flow and strengthening sections should lead to an increase in critical load. Therefore, in the absence of an experimental record of the complete q–δ diagram, with strength σ_t and fracture toughness K_{Ic} characteristics of a welded joint, in order to obtain a more conservative estimate of critical load it is simpler to proceed from the idealized curve considered in this work with one loss-of-strength section described by relationship (2.2) drawing upon (2.3).

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